**ICS 2022 Problem Sheet #2**

**Problem 2.1**: proof by contrapositive

Let x and y be real numbers, i.e., . If , then y ≤ x.

Proof:

We prove the contrapositive, if y x , then .

Assume y x. Since , then , which follows that . This finally leads to . □

**Problem 2.2**: proof by induction

Let n be a natural number with n ≥ 1. Prove that the following holds:

Proof:

We prove​  by induction

*Base case*:

We show that the equation is true for . Setting , the equation becomes

and hence the equation holds for .

*Induction step*:

Assume that the equation holds for some . Let's consider the case :

This shows that the equation holds for .

It follows by induction that

holds for arbitrary integers n.   □

**Problem 2.3**: sum of divisors in Haskell

1. Write a function divisors :: Int -> [Int] that returns the list of divisors of a given positive integer n. The list of divisors includes 1 and the number n itself.

divisors :: Int -> [Int]

divisors n = [ d | d <- [1..n], n `mod` d == 0 ]

1. Write a function sigma :: Int -> Int -> Int that takes the two arguments z and n and returns the sum of the zth powers of the positive divisors of n.
2. sigma :: Int -> Int -> Int

sigma z n = sum [ x^z | x <- [1..n], n `mod` x == 0 ]

or, using the divisors function:

1. sigma :: Int -> Int -> Int

sigma z n = sum [ x^z | x <- divisors n]